Design of 2 Hinged Steel Arch

P. L. Hein

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Design and general details
of two hinged steel arch

DESIGN AND GENERAL DETAILS
OF
TWO HINGED STEEL ARCH
SPAN 207' - 0" - C to C of END PINS.

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A Thesis presented by

Peter Leo Hein

to the

President and Faculty

of the

Armour Institute of Technology

for the Degree

of

Bachelor of Science in Civil Engineering

Having completed the prescribed course of study in

Civil Engineering.

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"Design and general details of two hinged plate girder arch, span two hundred and seven feet center to center of end pins ".

Loading:

The line load was assumed as 7000# per lineal foot of span. This heavy loading was assumed because the bridge is designed to carry heavy ore cars. The floor system was designed for Cooper Class E 50 Engine loading, with the exception of the stringers, which were designed to carry a load of 120000#, distributed equally on two pairs of driving wheels, spaced six feet center to center.

The dead load was determined by trial, and it amounted to 16200# per panel, per truss.

Computation of stresses:-

The center line of the arch ring is a parabola. This form was assumed because it is in perfect equilibrium under a load distributed uniformly horizontally over the entire span. If that is the case, we have no bending moment and no shear, but we will have a direct thrust to be taken by arch. In case we have eccentric loading , we will have bending moments, and it is desired to find what panel points must be loaded to give maximum positive and negative bending moments.

Place a single weight at I, as shown in fig. I. The condition of equilibrium is that ΣEF x DE= o. We know that the closing line of the polygon must pass through A &-B, because we have zero bending moment at A & B, therefore the only unknown is ce, which we will call yo. Now if yo is calculated for this load, we can construct the equilibrium polygon for the single load at I. Let the half span AK= KB = c, the rise at the crown equal k, the distance Kθ from mid span to single weight I = b, then the value of yo will be:

$$y_o = \frac{32}{5} k \times \frac{c^2}{5 c - b^2} \qquad \text{(Green)}$$

Now if b = nc where n is a fraction of the half span

$$(1) \quad y_o = \frac{32}{5 (- n^2)} k,$$

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Having found the general value of yo, we can determine the value of yo for a load at any point of the arch, hence we can draw the equilibrium polygon for any load. From the equilibrium polygon we can determine the value of H, by drawing lines from the extremity of the load line parallel to the sides of the equilibrium polygon. This value of H for any position of the load is found to be

$$(2) \quad H = \frac{1 - n^2}{2} \times \frac{5\,(5 - n^2)}{32} \times \frac{c}{k} \; w \text{ (Green)}$$

The span was divided into eighteen equal parts of 11' - 6" each, and the numerical values of the quantities in formula obtained. The load was placed at each panel point successively and the value of yo obtained by formula (1), and having the value of yo we can compute H from formula (2). Values of

$$(5 - n^2) \quad 5(5 - n^2) \quad \frac{32}{5(5 - n^2)} \quad 1/2\,(1 - n^2) \;\&\; 1/2(1 - n^2)$$

$$\div \frac{32}{5(5 - n^2)}$$

are given in table on page 13. Since n^2 is positive, whether n is positive or negative, the value of yo & H will be symetrical on each side of the crown of arch.

We may now compute the bending moment at different points, for a load on any one point. If y denotes the ordinate from the springing line (AB) to the equilibrium polygon, and z the ordinate to the parabola at the same point the bending moment is

$$M = H\,(y - z)$$

The ordinates y and z can be easily calculated. Thus if the load is placed at the third panel point from the end, yo is found from table I to be 1.4048 k,. Since the span is divided into eighteen equal divisions, the number of divisions on one side is three and on the other fifteen, therefore the values of y on one side are 1/3 and 2/3 of yo, while on the other side they are 1/15, 2/15, 3/15 ---- 14/15 of yo. Table No. 2 gives these values of y.

The ordinate z is proportional to the products of the segments into which it divides the span, this being a property of the parabola. These values of z are subtracted from the values of y and we obtain table No 2, which gives values of (y - z). The values of (y-z) are multiplied by H, and we obtain the quantity H(y-z), which as shown before, is the bending moment. The values of H(y-z) are found in table No 3.

To get the bending moment at any panel point of the arch for a single load on the span, multiply the constant in the table by the load at point of division and half the span. As for example, if we want to obtain the bending moment at the eight panel point due to a load on thirteen, multiply the constant found in the horizontal column thirteen, and in the vertical column eight, namely- .0493 by WC where W **is** the weight and C the half span.

To get the maximum positive or negative bendingmoments at any one point of division, add the positive or negative constants in a vertical column and multiply by WC.

The direct thrust in the arch due to the loading is:

$$T = \frac{H\ C\ W}{K} \qquad \text{(Green)}$$

Shear:-

Adhering still to the single weight, at a distance by from the middle of the span, we can solve for the reaction P2 by taking moments about the left abutment, hence, P2 is found to be $\frac{c+b}{2c}$ W. and $P_1 = \frac{c-b}{2c} W$

As the parabola is in equilibrium under a full uniform horizontally distributed load, we will have no **shear**, therefore, if we would construct a shear diagram, it would be the same as a beam under a uniform distributed load, as shown by the figure a i m, n & l in Fig. 1 If then we can determine the value of ai, we can draw this portion of the figure. Draw a tangent to the arch as AL, this tangent will intersect the middle ordinate at a distance 2k above the line AB, where k is rise of arch. Now if we draw the line o - 4 in the stress diagram, parallel to AL, it will intersect the load line at 4 and the distance 3 - 4 will be the necessary force combined with H to give a thrust coinciding with the **rib** at the springing point. Therefore lay off 3 - 4 at ai and at ln and connect the two points by a straight line in, the ordinate to this line from al at all the points will represent the amount of vertical force to be combined with the horizontal thrust to put the rib in equilibrium.

Formula for vertical Shear:

If y is the ordinate to in from any point of al and Y_1 its value at the springing, we have that

$$Y_1 : H :: 2k : c$$

because triangle, o, 4, 3 & A L K are similar. Therefore:

$$Y_1 = \frac{2k}{c} H$$

,and the vertical shear v in the web at the abutments on the left, will then be,

$$V = P_1 - Y_1 = \frac{c \pm b}{2c} W - \frac{2k}{c} H$$

For successive points P_1 will remain the value of the original shear until we pass the load, when it will become $P_1 - W$ or $- P_2$. Y will diminish at a constant rate; and subtracting y_1 from P_1, we get the value which multiplied by W gives the shear.

Tables No. 4 & 5 give values of P_1, P_2 and Y_1 for the loads at different panel points. Subtracting the values of Y_1 from P_1, we obtain table no 6, which gives values P-Y, and these constants multiplied by the load (W) at a panel point gives the vertical shear. To get the shear midway between the points of division two and three for a load at five we go to table No 6, and we find in the vertical column between two and three, and horizontal column five the number .2034, this number multiplied by W gives the shear. The maximum positive and negative shear at a section is found by adding the positive numbers and the negative numbers in one column respectively, and the positive numbers multiplied by W gives greatest positive shear, while the negative numbers multiplied by W gives the greatest negative shear.

Bending moment and direct thrust due to a change of temperature of an arch which is hinged at the ends, if exposed to a change of temperature, it will tend to change its shape. But as the abutments are fixed the arch cannot change its span, hence, the change will be manifested by a rise or fall of the crown of the arch. If the temperature changes, the arch tends to expand or contract in all directions, hence, the span would be lenghtened just in proportion to the rise in temperature t, the coefficient of expansion (e) and the length of the span 2c.

Therefore the change in length would be 2t ec, As the span is not allowed to change its length, we have a horizontal thrust Ht, acting at the abutments, which keeps arch in equilibrium. This thrust causes a bending moment and direct thrust throughout the arch ring. The value of this thrust Ht is

$$Ht = \frac{15}{8} \times \frac{t\ e'\ E\ I}{k^2} \qquad \text{(Green)}$$

In the calculation of the stresses E was assumed as 26.000,000, t = ~~E~~ ±75° F. e = .0000065 k= 46' = 552" & Ic = 300 000. This value of ~~k~~ Ic was found where the area of the flanges was greatest and used throughout the whole span, this would make temperature stresses at crown and abutments slightly larger than they actually are.

Having found the value of Ht, we can determine the bending moments of the different panel points by multiplying Ht by z, the distance from A B to center line of arch. The direct thrust of crown of arch is Ht, and at any other point it is equal to Ht sec θ, where θ is the angle that the tangent at the point in consideration makes with the horizontal.

Bending Moment and Direct Thrust due to shortening of rib.

The effect of placing a load on the arch is to shorten the rib, and as the span cannot change, this shortening of the rib will develope a force Hs. The value of Hs is $Hs = -\frac{15}{8}\ \frac{sI_c}{k^2}$ (Merriman). Where s is the unit compressive stress, Ic moment of inertia and k the rise of arch, s was taken as 10000, and Ic as 300 000. Having found Hs, we can determine the bending moment and direct thrust in the same way as the temperature stresses were determined, the only difference being that in changes of temperature, we can have either positive or negative bending moment, while for shortening of rib, we can only have positive moment.

Shear due to Change of Temperature.

The value of Ht having been found, we can determine a uniform load which would give the same value of Ht by the formula $Ht\ k = \frac{W\ c\ .\ 4.5}{11.5}$. Knowing the uniform distributed load we can calculate the reaction, namely

$$R = \frac{Wl}{2},$$

from this reaction we can determine the vertical shear in each panel. These values are both positive and negative, as the temperature falls or rises.

The same reasoning applies to the shear due to the shortening of the rib, except that for shortening of the rib, we can only have positive shear.

Dead Load Stresses

Since the dead load is approximately distributed uniform horizontally, it does not cause bending moment, but only a direct thrust. The value of H due to dead load is H = 4.5 c w which gives the thrust at crown, while the thrust at any other point is equalto $H \sec\theta$ where θ is the angle that the tangent at the point in consideration makes with the horizontal.

Wind Stresses.

The wind loading was assumed to be equal to 50# per square foot of exposed surface. This gave a wind load on the main arch rib varying from 9212# per panel at the crown to 11706# per panel at the springing line. These loads plus one half that which comes on the post was assumed to be taken by two latural systems, one in the place of the top flanges of arch and the other in the bottom flanges of arch. The wind on the train was 5750# per panel, and on the floor and stringers it was 1450# per panel, to this was added one half of load on posts and taken by a latteral system in the plane of the floor system and carried to the end post, then through the cross bracing at end to the abutments. The wind load on the side of arch would tend to overturn the bridge as a whole, and this would throw a load on the main arch rib, this load was determined, but it was so small compared with the dead load stress that it was neglected.

The stresses which are tabulated in the following pages were combined to give maximum compression and tension in the members, and these members which were subject to alternate strains were proportioned to resist each kind of strain. Both of the strains were however considered as increased by an amount equal to eight-tenths of the least of the two strains.

All the members in the bridge were designed according to Cooper's specification for Steel Railroad Bridges and Viaducts.

VALUES of H and Yo.

$n = \frac{b}{c}$	o	.1111	. 2222	. 3333	. 4444	.5556	.6667	.7778.	.8889
n^2	o	.0123	.0488	.1009	.1975	.3085	.4443	.6048	.7900
$5-n^2$	5.	4.9877	4.9512	4.8025	4.6915	4.5557	4.3952	4.3991	4.2100
$5(5-n^2)$	25	24.9385	24.7560	24.4955	24.0125	23.4575	22.7785	21.9760	21.0500
$\frac{32}{5(5-n^2)} =$	1.23	1.2853	1.2927	1.3094	1.3354	1.3462	1.4043	14561	1.5201

Multiply Values in Last Column by K to give Yo.

$1/2(1-n^2) =$	.5000	.4938	.4756	.4202	.3964.	.3457	.2778	.2976	.1050
$\frac{5(1-n^2)(5-n^2)}{2 \times 32} =$	,3906	.3877.	.3679	.3280	.	2969	.2534	.1978	.0691

Multiply by $\frac{c}{k}$ w to give H.

Direct Thrust due to Dead Load.

Formula: $$T = \frac{HCW \sec\theta}{K}$$

H - is horizontal thrust
C - is half span of arch
W - is load at panel point in pounds
F - is angle of inclination of member to horizontal
K - is rise of arch
T - is stress in both flanges

Panel	H	C	W	sec θ	K	T
9	4.5	103.5	16200	1.0012	46	164200
8	"	"	"	1.0109	"	165800
7	"	"	"	1.0214	"	167500
6	"	"	"	1.0715	"	175750
5	"	"	"	1.0946	"	179500
4	"	"	"	1.1345	"	186100
3	"	"	"	1.1924	"	195500
2	"	"	"	1.2334	"	203100
1	"	"	"	1.3062	"	213900

Stresses in flanges due to positive bending moment.

M-is constant found in table No3
W-is load at panel point in pounds
C-is half span of **Arch**
x-is effective depth of arch ring in feet
T-is stress per flange

Panel	M	C	W	x	T
9	.27308	103.5	40250	7	162500
8	.30354	"	"	"	180600
7	.39995	"	"	"	238000
6	.48652	"	"	"	289500
5	.57390	"	"	"	341500
4	.59117	"	"	"	351800
3	.55352	"	"	"	329400
2	.44989	"	"	"	264800
1	.26420	"	"	"	157200

Formula:- $$T = \frac{MCW}{x}$$

Direct thrust due to positive bending moment loading.

Formula $$T = \frac{HCW \sec \theta}{K}$$

H-is horizontal thrust
C-is half span of Arch
W-is load at panel point in pounds
θ-is angle of inclination of member to horizontal
k-is rise of Arch
T-is stress in both flanges.

Panel	H	C	W	Secθ	k	T
9	1.9022	103.5	40250	1.0012	46	172500
8	2.1587	"	"	1.0109	"	197600
7	2.4266	"	"	1.0214	"	224400
6	2.0369	"	"	1.0715	"	197700
5	2.0369	"	"	1.0946	"	201900
4	1.6429	"	"	1.1345	"	168800
3	1.2813	"	"	1.1924	"	138300
1	1.2813	"	"	1.3062	"	~~151500~~
2	1.2813	"	"	1.2384	"	~~143700~~

Stresses in flanges due to negative bending moment.

Formula $$T = \frac{MCW}{x}$$

M-is constant found in table No3
W-is load at panel point in pounds
C-is half span of Arch
x-is stress per flange

Panel	M	C	W	X	T
9	-.25750	103.5	40250	7	153200
8	-.29946	"	"	"	178100
7	-.33966	"	"	"	231800
6	-.52259	"	"	"	311000
5	-.56470	"	"	"	336000
4	-.59077	"	"	"	351200
3	-.55371	"	"	"	329500
2	-.44407	"	"	"	264300
1	-.26153	"	"	"	155600

Direct thrust due to positive bending moment Loading.

Formula :- $$T = \frac{HCW \sec \theta}{K}$$

H-is horizontal thrust
C-is half span of arch
W-isload at panel point in pounds
θ-is angle of inclination of member to horizontal
k-is rise of Arch
T-is stress in both flanges

Panel	H	C	W	sec θ	k	T
9	2.5613	103.5	40250	1.0012	46	232300
8	2.3048	"	"	1.0109	"	211100
7	2.0369	"	"	1.0214	"	188500
6	2.4266	"	"	1.0715	"	235400
5	2.4266	"	"	1.0946	"	240500
4	2.8143	"	"	1.1345	"	289200
3	3.1822	"	"	1.1924	"	343600
2	3.1822	"	"	1.2384	"	357000
1	3.1822	"	"	1.3062	"	376500

Stress in flanges due to change in temperature

Formula:- $Ht = \frac{15}{8} \times \frac{t\ e\ E\ I}{k^2}$

E-is 26,000,000 (modulus of elasticity)

t-is ±75° F (change of temperature)

e-is 0.000,0065 (coefficient of expansion)

k= 46' =552" rise of arch
Ic =300000 moment of inertia

Ht= 23400# horizontal thrust

$T = \frac{Ht\ z}{x}$

z - is distance from springingline to arch

x - is effective depth of arch ring in feet.

T stress per flange.

Panel	Ht	z	x	T
9	23400	46	7	153750
8	"	45.43	"	151850
7	"	43.75	"	146200
6	"	41.36	"	138250
5	"	36.91	"	123400
4	"	31.8	"	106300
3	"	25.65	"	85720
2	"	18.18	"	60770
1	"	9.66	"	32290

Note:-
The positive bending moment is the same as the negative, except that the sign of the stresses are changed.

Direct thrust due to a change of temperature.

T = Ht sec θ

T=Stress in both flanges
Ht= horizontal thrust
θ = The angle of inclination of member to horizontal

Panel	sec θ	Ht	T
9	1.0012	23400	23430
8	1.0109	"	23650
7	1.0214	"	23900
6	1.0715	"	25080
5	1.0946	"	25610
4	1.1345	"	26550
3	1.1924	"	27900
2	1.2384	"	28980
1	1.3062	"	30520

Stresses in flanges due to shortening of rib

Formula $$Hs = \frac{-15}{8} \times \frac{SIc}{k^2}$$

S = 10000 (unit compressive stress)

Ic= 300000 moment of inertia

k = 46' rise of Arch

H_s = 18500 " horizontal thrust

$$T = \frac{H_s Z}{x}$$

Z is distance from springing line to Arch

X is effective depth of arch ring in feet
T is stress per flange

Panel	z	Hs	X	T
9	46	18500	7	121400
8	45.43	"	"	120000
7	43.75	"	"	115500
6	41.36	"	"	109200
5	36.91	"	"	97440
4	31.8	"	"	33950
3	25.65	"	"	67700
2	13.18	"	"	48000
1	9.66	"	"	25500

Note:-
only a negative bending moment occurs due to shortening of rib.

Direct thrust due to shortening of rib

Formula $T = H_s \sec \theta$

Hs is horizontal thrust

θ is angle of inclination of member to horizontal

T is stress in both flanges

Panel	sec θ	Hs	T
9	1.0012	18500	18520
8	1.0109	"	18700
7	1.0214	"	18900
6	1.0715	"	19830
5	1.0946	"	20990
4	1.1345	"	20250
3	1.1924	"	22060
2	1.2384	"	22910
1	1.3062	"	24130

Positive shear due to line load

S = W N

W is load at panel point

N is constant found in table No6

S is shear

Panel	W	N	S
1	40250	2.3142	93140
2	"	1.8695	75240
3	"	1.5254	61400
4	"	~~135.~~91 1.3591	54700
5	"	1.3724	55220
6	"	1.6033	64530
7	"	1.8990	76830
8	"	1.0979	84430
9	"	1.2303	89760
10	"	2.2251	89560
11	"	2.0743	83480
12	"	1.8774	75560
13	"	1.5737	63340
14	"	1.3302	53540
15	"	1.3137	52880
16	"	1.4714	59220
17	"	1.8117	72920
18	"	2.3452	94400.

negative shear due to line load

S = W N

W is load at panel point
N is constant found in table No6
S is shear

Panel	W	N	S
1	40250	2.3452	94400
2	"	1.8117	72920
3	"	1.4714	59220
4	"	1.3137	52880
5	"	1.3302	53540
6	"	1.5737	63340
7	"	1.8774	75560
8	"	2.0743	83480
9	"	2.2251	89560
10	"	2.2303	89760
11	"	2.0979	84430
12	"	1.8990	76830
13	"	1.6033	64530
14	"	1.3774	55440
15	"	1.3591	54700
16	"	1.5254	61400
17	"	1.8695	75240
18	"	2.3142	93140

Shear due to change of temperature.
Hk = WCX4.5

W = 2484

R = θ = 22350

Panel	S
1	21114
2	18630
3	16146
4	13662
5	11178
6	8694
7	6210
8	3726
9	1242

These are both positive and negative values because the positive shear is the same as the negative.

Shear due to shortening of rib

W = 1831 R = X = 16479

Panel	S
1	15572
2	13740
3	11908
4	10076
5	8244
6	6412
7	4580
8	2748
9	916

we can have only a positive shear due to shortening of rib.

No.1.

	1	16	17
1	1.52	.1788	.0894
2	.72	.1820	.0910
3	.46	.1874	.0937
4	.341	.1948	.0974
5	.26	.2054	.1027
6	.21	.2182	.1091
7	.184	.2350	.1175
8	.160	.2570	.1285
9	.14	.2844	.1422
10	.12	.3214	.1607
11	.117	.3694	.1847
12	.10	.4364	.2182
13	.102	.5342	.2671
14	.09	.6820	.3410
15	.09	.9364	.4682
16	.09	1.4561	.7280
17	.08	1.4304	1.5202
Z	.210	.3952	.2100

Arch.

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No 1

P.L. Hein

VALUES of Y_o

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1.5202	1.4304	1.3410	1.2516	1.1622	1.0728	.9834	.8940	.8046	.7152	.6258	.5364	.4470	.3576	.2682	.1788	.0894
2	.7280	1.4561	1.3650	1.2840	1.1830	1.0920	1.0010	.9100	.8190	.7280	.6370	.5460	.4550	.3640	.2730	.1820	.0910
3	.4682	.9364	1.4048	1.3118	1.2181	1.1244	1.0307	.9370	.8433	.7496	.6559	.5622	.4685	.3748	.2811	.1874	.0937
4	.3410	.6820	1.0203	1.3642	1.2662	1.1688	1.0714	.9740	.8766	.7792	.6818	.5844	.4870	.3896	.2922	.1948	.0974
5	.2671	.5342	.8013	1.0684	1.3354	1.2324	1.1297	1.0270	.9243	.8216	.7189	.6162	.5135	.4108	.3081	.2054	.1027
6	.2182	.4364	.6546	.8728	1.0910	1.3094	1.2000	1.0910	.9819	.8728	.7637	.6546	.5455	.4364	.3273	.2182	.1091
7	.1847	.3694	.5541	.7388	.9235	1.1082	1.2927	1.1750	1.0575	.9400	.8225	.7050	.5875	.4700	.3525	.2350	.1175
8	.1607	.3214	.4821	.6428	.8035	.9642	1.1249	1.2853	1.1565	1.0280	.8995	.7710	.6425	.5140	.3855	.2570	.1285
9	.1422	.2844	.4266	.5688	.7110	.8532	.9954	1.1376	1.2800	1.1376	.9954	.8532	.7110	.5688	.4266	.2844	.1422
10	.1285	.2570	.3855	.5140	.6425	.7710	.8995	1.0280	1.1565	1.2853	1.1249	.9642	.8035	.6428	.4821	.3214	.1607
11	.1175	.2350	.3525	.4700	.5875	.7050	.8225	.9400	1.0575	1.1750	1.2927	1.1082	.9235	.7388	.5541	.3694	.1847
12	.1091	.2182	.3273	.4364	.5455	.6546	.7637	.8728	.9819	1.0910	1.2000	1.3094	1.0910	.8728	.6546	.4364	.2182
13	.1027	.2054	.3081	.4108	.5135	.6162	.7189	.8216	.9243	1.0270	1.1297	1.2324	1.3354	1.0684	.8013	.5342	.2671
14	.0974	.1948	.2920	.3896	.4870	.5844	.6818	.7792	.8766	.9740	1.0714	1.1688	1.2662	1.3642	1.0230	.6820	.3410
15	.0937	.1874	.2811	.3748	.4685	.5622	.6559	.7496	.8433	.9370	1.0307	1.1244	1.2181	1.3118	1.4048	.9364	.4682
16	.0910	.1820	.2730	.3640	.4550	.5460	.6370	.7280	.8190	.9100	1.0010	1.0920	1.1830	1.2840	1.3650	1.4561	.7280
17	.0894	.1788	.2682	.3576	.4470	.5364	.6258	.7152	.8046	.8940	.9834	1.0728	1.1622	1.2516	1.3410	1.4304	1.5202
Z	.2100	.3952	.5575	.6915	.8025	.8991	.9512	.9877	1.0000	.9877	.9512	.8991	.8025	.6915	.5575	.3952	.2100

Note:–

Horizontal row of figures 1, 2,–17 are points of division of Arch
Vertical " " " " " – " gives location of load on Arch
Y_o is distance from springing line to equilibrium polygon
Z " " " " " " Arch Ring

No 1

P.L. Hein

No. 2

	1	16	17
1	1.3102	-.2164	-.1206
2	.5180	-.2132	-.1190
3	.2582	-.2078	-.1163
4	.1310	-.2004	-.1126
5	.0571	-.1898	-.1073
6	.0082	-.1770	-.1009
7	-.0253	-.1602	-.0925
8	-.0493	-.1382	-.0815
9	-.0678	-.1108	-.0678
10	-.0815	-.0738	-.0493
11	-.0925	-.0258	-.0253
12	-.1009	.0412	.0082
13	-.1073	.1390	.0571
14	-.1126	.2868	.1310
15	-.1163	.5412	.2582
16	-.1193	1.0609	.5180
17	-.1206	1.0352	1.3102

No. 2

P.L. Hein.

VALUES of (y-z) $M = H(Y-Z)$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1.3102	1.0352	.7835	.5601	.3597	.1737	.0322	-.0937	-.1954	-.2725	-.3254	-.3627	-.3555	-.3339	-.2893	-.2164	-.1206
2	.5180	1.0609	.8075	.5924	.3805	.1929	.0498	-.0777	-.1810	-.2597	-.3142	-.3531	-.3475	-.3275	-.2845	-.2132	-.1190
3	.2582	.5412	.8473	.6203	.4156	.2253	.0795	-.0507	-.1567	-.2381	-.2953	-.3369	-.3340	-.3167	-.2764	-.2078	-.1163
4	.1310	.2868	.4655	.6727	.4637	.2697	.1202	-.0137	-.1234	-.2085	-.2694	-.3147	-.3155	-.3019	-.2653	-.2004	-.1126
5	.0571	.1390	.2438	.3769	.5329	.3333	.1785	.0393	-.0757	-.1661	-.2323	-.2829	-.2890	-.2807	-.2494	-.1898	-.1073
6	.0082	.0412	.0971	.1813	.2885	.4103	.2488	.1033	-.0181	-.1149	-.1875	-.2445	-.2570	-.2551	-.2302	-.1770	-.1009
7	-.0253	-.0258	-.0034	.0473	.1210	.2091	.3415	.1873	.0575	-.0477	-.1287	-.1941	-.2150	-.2215	-.2050	-.1602	-.0925
8	-.0493	-.0738	-.0754	-.0487	.0010	.0651	.1737	.2976	.1565	.0403	-.0517	-.1281	-.1600	-.1775	-.1720	-.1382	-.0815
9	-.0678	-.1108	-.1309	-.1227	-.0915	-.0459	.0442	.1499	.2800	.1499	.0442	-.0459	-.0915	-.1227	-.1309	-.1108	-.0678
10	-.0815	-.1382	-.1720	-.1775	-.1600	-.1281	-.0517	.0403	.1565	.2976	.1737	.0651	.0010	-.0487	-.0754	-.0738	-.0493
11	-.0925	-.1602	-.2050	-.2215	-.2150	-.1941	-.1287	-.0477	.0575	.1873	.3415	.2091	.1210	.0473	-.0034	-.0258	-.0253
12	-.1009	-.1770	-.2302	-.2551	-.2570	-.2445	-.1875	-.1149	-.0181	.1033	.2488	.4103	.2885	.1813	.0971	.0412	.0082
13	-.1073	-.1898	-.2494	-.2807	-.2890	-.2829	-.2323	-.1661	-.0757	.0393	.1785	3333	.5329	.3769	.2438	.1390	.0571
14	-.1126	-.2004	-.2653	-.3019	-.3155	-.3147	-.2694	-.2085	-.1234	-.0137	.1202	.2697	.4637	.6727	.4655	.2868	.1310
15	-.1163	-.2078	-.2764	-.3167	-.3340	-.3369	-.2953	-.2381	-.1567	-.0507	.0795	.2253	.4156	.6203	.8473	.5412	.2582
16	-.1193	-.2132	-.2845	-.3275	-.3475	-.3531	-.3142	-.2597	-.1810	-.0777	.0498	.1929	.3805	.5925	.8075	1.0609	.5180
17	-.1206	-.2164	-.2893	-.3339	-.3555	-.3627	-.3254	-.2725	-.1954	-.0937	.0322	.1737	.3597	.5601	.7835	1.0352	1.3102

No 2

P.L. Hein.

No. 3

W.on.	1.	7	Value of H
17.	-.0083	00	$.06907 \frac{c}{h} W$
16.	-.0161	03	.13750 "
15.	-.0230	11	.19775 "
14.	-.0285	32	.25341 "
13.	-.0319	70	.29686 "
12.	-.0331	27	.32802 "
11.	-.0340	93	.36794 "
10.	-.0316	91	.38770 "
9.	-.0265	265	.39060 "
8.	-.0191	316	.38770 "
7.	-.0093	40	.36974 "
6.	.0027	31	.32802 "
5.	.0170	19	.29686 "
4.	.0332	85	.25341 "
3	.0511	30	.19775 "
2.	.0703	62	.13570 "
1.	.0900	83	.06907 "

No 3

P.L. Hein.

VALUES of M B.M. = MWC

W.on.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14	15.	16.	17	Value of H
17.	-.0083	-.0150	-.0200	-.0231	-.0249	-.0251	-.0225	-.0188	-.0135	-.0065	.0022	.0120	.0255	.0387	.0541	.0713	.0900	.06907 $\frac{c}{k}$W
16.	-.0161	-.0289	-.0386	-.0444	-.0458	-.0479	-.0426	-.0352	-.0246	-.0106	.0068	.0259	.0530	.0804	.1096	.1440	.0703	.13750 "
15.	-.0230	-.0411	-.0546	-.0626	-.0641	-.0666	-.0584	-.0471	-.0310	-.0100	.0157	.0446	.0841	.1228	.1676	.1071	.0511	.19775 "
14.	-.0285	-.0508	-.0672	-.0765	-.0775	-.0797	-.0683	-.0528	-.0313	-.0035	.0305	.0683	.1200	.1705	.1180	.0727	.0332	.25341 "
13.	-.0319	-.0564	-.0740	-.0833	-.0830	-.0840	-.0690	-.0493	-.0225	.0116	.0530	.0989	.1611	.1019	.0724	.0413	.0170	.29686 "
12.	-.0331	-.0581	-.0755	-.0837	-.0811	-.0802	-.0615	-.0377	-.0059	.0339	.0816	.1346	.0978	.0595	.0318	.0135	.0027	.32802 "
11.	-.0340	-.0589	-.0754	-.0815	-.0755	-.0714	-.0474	-.0179	.0212	.0689	.1256	.0769	.0481	.0174	-.0013	-.0095	-.0093	.36794 "
10.	-.0316	-.0536	-.0667	-.0688	-.0583	-.0498	-.0201	.0156	.0607	.1154	.0673	.0252	.0041	-.0189	-.0292	-.0286	-.0191	.38770 "
9.	-.0265	-.0433	-.0511	-.0479	-.0319	-.0179	.0173	.0586	.1094	.0586	.0173	-.0179	-.0319	-.0479	-.0511	-.0433	-.0265	.39060 "
8.	-.0191	-.0286	-.0292	-.0189	.0041	.0253	.0675	.1154	.0607	.0156	-.0200	-.0497	-.0583	-.0688	-.0667	-.0536	-.0316	.38770 "
7.	-.0093	-.0095	-.0013	.0174	.0481	.0769	.1256	.0689	.0212	-.0179	-.0474	-.0714	-.0755	-.0815	-.0754	-.0589	-.0340	.36974 "
6.	.0027	-.0135	.0318	.0595	.0978	.1346	.0816	.0339	-.0059	-.0377	-.0615	-.0802	-.0811	-.0837	-.0755	-.0581	-.0331	.32802 "
5.	.0170	.0413	.0724	.1019	.1611	.0989	.0530	.0116	-.0225	-.0493	-.0690	-.0840	-.0829	-.0833	-.0740	-.0564	-.0319	.29686 "
4.	.0332	.0727	.1180	.1705	.1200	.0683	.0305	-.0035	-.0313	-.0528	-.0683	-.0797	-.0775	-.0765	-.0672	-.0508	-.0285	.25341 "
3	.0511	.1071	.1676	.1228	.0841	.0446	.0157	-.0100	-.0310	-.0471	-.0584	-.0666	-.0641	-.0626	-.0546	-.0411	-.0230	.19775 "
2	.0703	.1440	.1096	.0804	.0529	.0259	.0068	-.0106	-.0246	-.0352	-.0426	-.0479	-.0458	-.0444	-.0386	-.0289	-.0162	.13570 "
1	.0900	.0713	.0541	.0387	.0255	.0120	.0022	-.0065	-.0135	-.0188	-.0225	-.0251	-.0239	-.0231	-.0200	-.0150	-.0083	.06907 "

Note:-

k = Rise of Arch

W = Load at panel point

2C = Span of Arch

B.M = Bending Moment.

No 3

P.L. Hein.

No.4

P_1	1	17	18	P_2
1	.9445	55	-.0555	1
2	.8889	11	-.1111	2
3	.8334	66	-.1666	3
4	.7778	22	-.2222	4
5	.7222	78	-.2778	5
6	.6667	33	-.3333	6
7	.6111	89	-.3889	7
8	.5556	44	-.4444	8
9	.5000	00	-.5000	9
10	.4444	56	-.5556	10
11	.3889	11	-.6111	11
12	.3333	67	-.6667	12
13	.2778	22	-.7222	13
14	.2222	78	-.7778	14
15	.1666	34	-.8334	15
16	.1111	89	-.8889	16
17	.0555	55	-.9445	17

No 4

P. L. Hein.

VALUES of P_1 and P_2

P_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	P_2
1	.9445	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	-.0555	1
2	.8889	.8889	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	-.1111	2
3	.8334	.8334	.8334	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	-.1666	3
4	.7778	.7778	.7778	.7778	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	-.2222	4
5	.7222	.7222	.7222	.7222	.7222	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	-.2778	5
6	.6667	.6667	.6667	.6667	.6667	.6667	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	-.3333	6
7	.6111	.6111	.6111	.6111	.6111	.6111	.6111	-.3889	-.3889	-.3889	-.3889	-.3889	-.3889	-.3889	-.3889	-.3889	-.3889	-.3889	7
8	.5556	.5556	.5556	.5556	.5556	.5556	.5556	.5556	-.4444	-.4444	-.4444	-.4444	-.4444	-.4444	-.4444	-.4444	-.4444	-.4444	8
9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	9
10	.4444	.4444	.4444	.4444	.4444	.4444	.4444	.4444	.4444	.4444	-.5556	-.5556	-.5556	-.5556	-.5556	-.5556	-.5556	-.5556	10
11	.3889	.3889	.3889	.3889	.3889	.3889	.3889	.3889	.3889	.3889	.3889	-.6111	-.6111	-.6111	-.6111	-.6111	-.6111	-.6111	11
12	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	-.6667	-.6667	-.6667	-.6667	-.6667	-.6667	12
13	.2778	.2778	.2778	.2778	.2778	.2778	.2778	.2778	.2778	.2778	.2778	.2778	.2778	-.7222	-.7222	-.7222	-.7222	-.7222	13
14	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	.2222	-.7778	-.7778	-.7778	-.7778	14
15	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	.1666	-.8334	-.8334	-.8334	15
16	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	.1111	-.8889	-.8889	16
17	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	.0555	-.9445	17

No 4

P. L. Hein.

No. 5

		17	18	
1	.1305	-1151	-.1305	1
2	.2563	-2261	-.2563	2
3	.3735	-3296	-.3735	3
4	.4787	4223	-.4787	4
5	.5607	-4948	-.5607	5
6	.6196	-5467	-.6196	6
7	.6950	-6312	-.6950	7
8	.7323	-6462	-.7323	8
9	.7378	-6510	-.7378	9
10	.7323	-6462	-.7323	10
11	.6950	-6132	-.6950	11
12	.6196	-5467	-.6196	12
13	.5607	-4948	-.5607	13
14	.4787	-4223	-.4787	14
15	.3735	-3296	-.3735	15
16	.2563	-2261	-.2563	16
17	.1305	-1151	-.1305	17

No 5.

P.L. Hein.

VALUES of Y

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	.1305	.1151	0997	0843	.0689	.0535	.0381	.0227	.0073	-.0073	-.0227	-.0381	-.0535	-.0689	-.0843	-.0997	-.1151	-.1305	1
2	.2563	.2261	.1959	.1657	.1355	.1053	.0751	.0449	.0147	-.0147	-.0449	-.0751	-.1053	-.1355	-.1657	-.1959	-.2261	-.2563	2
3	.3735	.3296	.2857	.2417	.1978	.1538	.1099	.0660	.0220	-.0220	-.0660	-.1099	-.1538	-.1978	-.2417	-.2857	-.3296	-.3735	3
4	.4787	.4223	.3660	.3097	.2534	.1971	.1407	.0844	.0281	-.0281	-.0844	-.1407	-.1971	-.2534	-.3097	-.3660	-.4223	-.4787	4
5	.5607	.4948	.4288	.3628	.2968	.2308	.1649	.0989	.0329	-.0329	-.0989	-.1649	-.2308	-.2968	-.3628	-.4288	-.4948	-.5607	5
6	.6196	.5467	.4738	.4009	.3280	.2551	.1822	.1093	.0364	-.0364	-.1093	-.1822	-.2551	-.3280	-.4009	-.4738	-.5467	-.6196	6
7	.6950	.6132	.5315	.4497	.3680	.2862	.2044	.1227	.0409	-.0409	-.1227	-.2044	-.2862	-.3680	-.4497	-.5315	-.6312	-.6950	7
8	.7323	.6462	.5600	.4738	.3877	.3015	.2154	.1292	.0430	-.0430	-.1292	-.2154	-.3015	-.3877	-.4738	-.5600	-.6462	-.7323	8
9	.7378	.6510	.5642	.4774	.3906	.3038	.2170	.1302	.0434	-.0434	-.1302	-.2170	-.3038	-.3906	-.4774	-.5642	-.6510	-.7378	9
10	.7323	.6462	.5600	.4738	.3877	.3015	.2154	.1292	.0430	-.0430	-.1292	-.2154	-.3015	-.3877	-.4738	-.5600	-.6462	-.7323	10
11	.6950	.6132	.5315	4497	.3680	.2862	.2044	.1227	.0409	-.0409	-.1227	-.2044	-.2862	-.3680	-.4497	-.5315	-.6132	-.6950	11
12	.6196	.5467	.4738	.4009	.3280	.2551	.1822	.1093	.0364	-.0364	-.1093	-.1822	-.2551	-.3280	-.4009	-.4738	-.5467	-.6196	12
13	.5607	.4948	.4288	.3628	.2968	.2308	.1649	.0989	.0329	-.0329	-.0989	-.1649	-.2308	-.2968	-.3628	-.4288	-.4948	-.5607	13
14	.4787	.4223	.3660	.3097	.2534	.1971	.1407	.0844	.0281	-.0281	-.0844	-.1407	-.1971	-.2534	-.3097	-.3660	-.4223	-.4787	14
15	.3735	.3296	.2857	.2417	.1978	.1538	.1099	.0660	.0220	-.0220	-.0660	-.1099	-.1538	-.1978	-.2417	-.2857	-.3296	-.3735	15
16	.2563	.2261	.1959	.1657	.1355	.1053	.0751	.0449	.0147	-.0147	-.0449	-.0751	-.1053	-.1355	-.1657	-.1959	-.2261	-.2563	16
17	.1305	.1151	.0997	.0843	.0689	.0535	.0381	.0227	.0073	-.0073	-.0227	-.0381	-.0535	-.0689	-.0843	-.0997	-.1151	-.1305	17

No 5.

P.L. Hein.

No.6.

W on	
17	-.07
16	-.14
15	-.20
14	-.25
13	-.28
12	-.28
11	-.30
10	-.28
9	-.23
8	-.17
7	-.08
6	.04
5	.16
4	.29
3	.45
2	.63
1.	.71

17	18	
06	-.7140	17
28	-.6326	16
38	-.4599	15
55	-.2991	14
74	-.1615	13
00	-.0471	12
21	.0839	11
06	.1767	10
10	.2378	9
18	.2879	8
43	.3061	7
34	.2863	6
70	.2829	5
01	.2565	4
30	.2069	3.
50	.1452	2.
96	.0750	1.

No 6.

P.L. Hein.

VALUES of N S = WN

W on	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
17	-.0750	-.0596	-.0442	-.0288	-.0134	.0020	.0174	.0328	.0482	.0628	.0782	.0936	.1090	.1244	.1398	.1552	.1706	-.7140	17
16	-.1452	-.1150	-.0848	-.0546	-.0244	.0058	.0360	.0662	.0964	.1258	.1560	.1862	.2164	.2466	.2768	.3070	-.6628	-.6326	16
15	-.2069	-.1630	-.1191	-.0751	-.0312	.0128	.0567	.1006	.1440	.1886	.2326	.2765	.3204	.3644	.4083	-.5477	-.5038	-.4599	15
14	-.2565	-.2001	-.1438	-.0875	-.0312	.0251	.0815	.1378	.1941	.2503	.3066	.3629	.4193	.4756	-.4681	-.4118	-.3555	-.2991	14
13	-.2829	-.2170	-.1510	-.0850	-.0190	.0470	.1129	.1789	.2449	.3107	.3767	.4427	.5086	-.4254	-.3594	-.2934	-.2274	-.1615	13
12	-.2863	-.2134	-.1405	-.0676	.0053	.0782	.1511	.2240	.2967	.3697	.4126	.5155	-.4116	-.3387	-.2658	-.1929	-.1200	-.0471	12
11	-.3061	-.2243	-.1426	-.0608	.0209	.1027	.1845	.2662	.3480	.4298	.5116	-.4067	-.3249	-.2431	-.1614	-.0796	.0021	.0839	11
10	-.2879	-.2018	-.1156	-.0294	.0567	.1429	.2290	.3152	.4014	.4874	-.4264	-.3402	-.2541	-.1679	-.0818	.0044	.0906	.1767	10
9	-.2378	-.1510	-.0642	.0226	.1194	.1962	.2830	.3698	.4566	-.4566	-.3698	-.2830	-.1962	-.1194	-.0226	.0642	.1510	.2378	9
8	-.1767	-.0906	-.0044	.0818	.1679	.2541	.3402	.4264	-.4874	-.4014	-.3152	-.2290	-.1429	-.0567	.0294	.1156	.2018	.2879	8
7	-.0839	-.0021	.0796	.1614	.2431	.324	.4067	-.5116	-.4298	-.3480	-.2662	-.1845	-.1027	-.0209	.0608	.1426	.2243	.3061	7
6	.0471	.1200	.1929	.2659	.3387	.4116	-.5155	-.4126	-.3697	-.2967	-.2240	-.1511	-.0782	-.0053	.0676	.1405	.2134	.2863	6
5	.1615	.2274	.2934	.3594	.4254	-.5086	-.4427	-.3767	-.3107	-.2449	-.1789	-.1129	-.0470	.0190	.0850	.1510	.2170	.2829	5
4	.2991	.3555	.4118	.4681	-.4756	-.4193	-.3629	-.3066	-.2503	-.1941	-.1378	-.0815	-.0251	.0312	.0875	.1438	.2001	.2565	4
3	.4599	.5038	.5477	-.4083	-.3644	-.3204	-.2765	-.2326	-.1886	-.1440	-.1006	-.0567	-.0128	.0312	.0751	.1191	.1630	.2069	3
2	.6326	.6628	-.3070	-.2768	-.2466	-.2164	-.1862	-.1560	-.1258	-.0964	-.0662	-.0360	-.0055	.0244	.0546	.0848	.1150	.1452	2
1	.7140	-.1706	-.1552	-.1398	-.1244	-.1090	-.0936	-.0782	-.0628	-.0482	-.0328	-.0174	-.0020	.0134	.0288	.0442	.0596	.0750	1

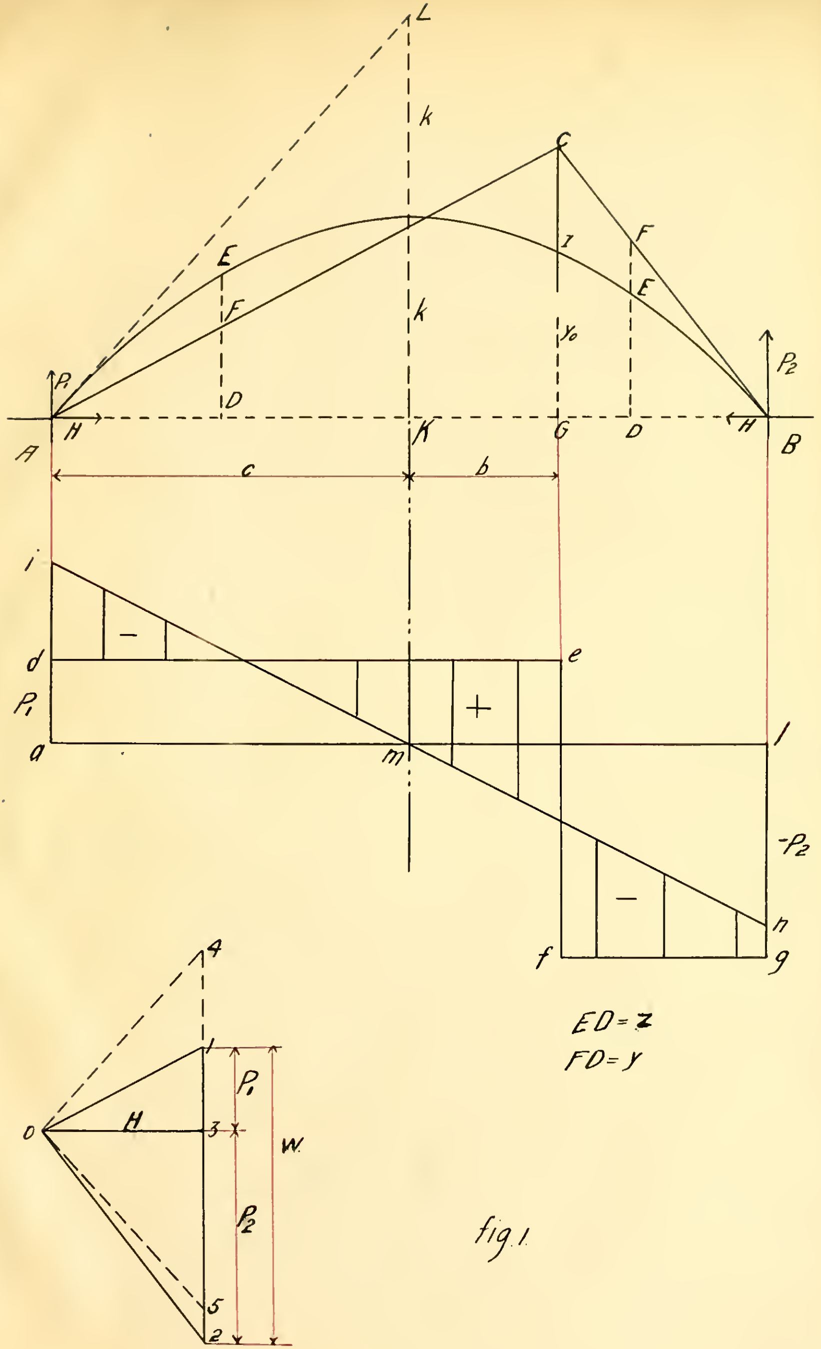
Note:—

W = Load at panel point

S = Shear.

No 6

P.L. Hein.


L
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h
f
g
4
1
P_1
H
0
3
W
P_2
5
2

$ED = z$

$FD = y$

fig. 1.